GCE

Mathematics

Advanced GCE

Unit **4730:** Mechanics 3

Mark Scheme for June 2011

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1	$[5\cos\theta - 4 = 0]$ $\cos\theta = 0.8$ $[I = 0.3(5\sin\theta - 0)$ or $\sin\theta = I \div (0.3 \times 5)]$ $I = 0.9$	M1 A1 M1 A1 [4]	For using $v_x - u_x = 0$ **or** for a triangle sketched with sides $I/0.3$, 4 and 5 with angles θ and 90^o opposite I/m and 5 respectively. AG For using $I = m(\Delta v)$ in 'y' direction or $I = \sqrt{\left((0.3\times5)^2 - (0.3\times4)^2\right)}$ M1

2 i	$(1.8 + 3.2)R_B = (3.2 + 0.9)\text{x}300 + 1.6\text{x}400$ Force exerted on AB is 374 N Force exerted on AC is 326 N	M1 A1 A1 B1 [4]	For taking moments about C for the whole for M1 need 3 terms; allow 1 sign error and/or 1 length error and/or still including sin/cos or for taking moments about B for whole $(1.8 + 3.2)R_C = (1.8 + 1.6)\text{x}400 + 0.9\text{x}300$ giving force on AC first: M1A1A1A1
ii	$0.9\text{x}300 + 1.2T = 1.8\text{x}374$ Tension is 336 N	M1 A1 A1 [3]	For taking moments about A for AB for M1 need 3 terms, allow 1 sign error and/or 1 length error and/or still including sin/cos or moments about A for AC $1.6\text{x}400 + 1.2T = 3.2\text{x}326$
iii	Horizontal component is 336 N to the left $[Y = 374 - 300]$ Vertical component is 74 N downwards	B1ft M1 A1ft [3]	For resolving forces on AB vertically

Give credit for part (ii) done on the way to part (i) if not contradicted in (ii).

3 i	$0.25(dv/dt) = -0.2v^2$ $0.25\int v^{-2}dv = -0.2t(+C)$ $-v^{-1}/4 = -t/5 + C$ [$1/4v = t/5 + 1/20$] $v = \dfrac{5}{4t+1}$ oe	M1 dep M1 A1 M1 A1 [5]	For using Newton's second law with $a = dv/dt$. Allow sign error and/or omitting mass For separating variables and attempting to integrate (ie get v^{-1} and t). For using $v(0) = 5$ to obtain C
ii	$x = (5/4)\ln(4t + 1)$ $(+ B)$ Subst $v = 0.2$ in (i) to find t Obtain $x(6)$ ($= 1.25 \ln 25$ oe (4.02359...)) Average speed is 0.671 ms^{-1}	M1 A1 M1 M1 A1 [5]	For using $v = dx/dt$ and integrating Implied by $t = 6$ May be written as $\frac{5}{12}\ln 5$
	Alternatively $\ln v = -0.8x + B$ Subst $v = 0.2$ in (i) to find t Obtain $x(0.2)$ ($= 1.25 \ln(5/0.2)$ oe (4.0239...)) Average speed is 0.671 ms^{-1}	M1 A1 M1 M1 A1	For using $mv(dv/dx) = -0.2v^2$, separating variables and integrating. Allow sign error and/or omitting mass. Implied by $t = 6$ May be written as $\frac{5}{12}\ln 5$

4 i	[$-0.2\text{x}2\ddot{\theta} = 0.2g\sin\theta$] $\dfrac{d^2\theta}{dt^2} = -4.9\sin\theta$ For small θ, $\sin\theta \approx \theta$ **and** $\ddot{\theta} = -4.9\theta$ represents SHM	M1 A1 B1 [3]	For using Newton's second law transversely. Allow sign error and/or sin/cos error and/or missing 0.2, g or l. AG
ii	$\theta = 0.15\cos(\sqrt{4.9}\,t)$ oe $t = 1.04$ at first occasion $t = 1.80$ at second occasion	M1 A1 A1 M1 A1 [5]	For using $\theta = A\cos(nt)$ or $A\sin(nt + \varepsilon)$. Allow sin/cos confusion for using $t_1 + t_2 = 2\pi / n$
iii	Angular speed is (-) $0.297 \text{ rads s}^{-1}$ Linear speed is (-) 0.594ms^{-1}	M1 A1 A1ft [3]	For using $\dot{\theta} = -An\sin(nt)$ oe. Allow sign error and/or ft from θ in (ii).

In (ii) & (iii) allow M marks if angular displacement/speed has been confused with linear.

5 i	[$\sin\gamma = 0.96 \div 1.2$] $\sin\gamma = 0.8$	M1 A1 [2]	For using $v_B\sin\gamma = u_B\sin\beta$
ii	$(m)2 - (m)u_B\cos\beta = (m)v_B\cos\gamma$ $2 = v_B(0.6 + 0.28 \div 1.2)$ $v_B = 2.4$, $u_B = 2$	M1 A1 M1 A1 A1 [5]	For using the principle of conservation of momentum. Allow sign error and/or $u_A\cos\alpha$ (instead of 2) for M1. allow $u_A\cos\alpha$ (instead of 2) for A1 For eliminating u_B or v_B. Allow with cos Or $2 = 0.28u_B + 0.72u_B$
iii	[$(2 + u_B\cos\beta)e = v_B\cos\gamma$] $(2 + 2 \text{ x } 0.28)e = 2.4 \text{ x } 0.6$ $e = \frac{9}{16}$ or 0.5625	M1 A1ft A1 [3]	For applying Newton's exp'tal law. Allow sign error and/or $u_A\cos\alpha$ (instead of 2) for M1. ft u_B and v_B only
iv	[$(y\text{-component})^2 = 13 - 4$] $v_A = (y\text{-component})_{\text{before}} = 3$	M1 A1 [2]	For using ½ $(m)v^2 = 6.5(m)$ **and** $(y\text{-component})^2 = v^2 - 2^2$. Allow 1 slip.

6 i	PE gain = $6\text{x}0.8(\sqrt{3}/2 - 1/\sqrt{2})$ $= 2.4(\sqrt{3} - \sqrt{2})$ EE loss = $\frac{9}{2(\pi/10)}[(0.8\pi/4 - \pi/10)^2 - (0.8\pi/6 - \pi/10)^2]$ EE loss = $45\pi[(0.2 - 0.1)^2 - (0.4 - 0.3)^2 \div 9]$ $= 5\pi(9\text{x}0.01 - 0.01) = 40\pi/100 = 0.4\pi$ J	M1 A1 M1 A1 A1 [5]	For using PE gain = $W(h_Y - h_X)$ Shown fully, with no slips AG For using EE loss = $\lambda(e_X^2 - e_Y^2)/2l$. Allow slips for M1. Fully correct No slips in simplification AG
ii	$T = 9(0.8\pi/6 - \pi/10) \div (\pi/10)$ $W\sin\theta - T = 6\times\sin(\pi/6) - 90\times(0.2 \div 6) = 0$ ➔ transverse acceleration is zero ½ $(6/9.8)v^2 = 0.4\pi - 2.4(\sqrt{3} - \sqrt{2})$ Maximum speed is 1.27 ms^{-1}	B1 M1 A1 M1 A1 A1 [6]	 For attempting to show that $W\sin\theta - T = 0$ at Y by subst $\theta = \pi/6$ AG No slips For using KE gain = EE loss – PE gain at Y. Need 3 terms, allow sign errors and/or g omitted.

7 i	$\frac{1}{2}mv^2 = \frac{1}{2}m5.6^2 - mg0.8(1 - \cos\theta)$ $v^2 = 15.68(1 + \cos\theta)$	M1 A1 A1	For using the principle of conservation of energy. Allow sign error, sin/cos; need 3 terms. AG No slips
	$T - mg\cos\theta = mv^2/r$	M1 A1	For using Newton's second law. Allow sign error and/or sin/cos and/or m omitted
	$[T - 0.3g\cos\theta = 0.3\text{x}15.68(1 + \cos\theta)/0.8]$ Tension is $2.94(3\cos\theta + 2)$ N oe	M1 A1 [7]	For substituting for v^2
ii	θ is 131.8° (or 2.3 rads) Accept 132° (exact) v is 2.29	M1 A1 B1 [3]	For putting $T = 0$ and attempting to solve accept $\theta = \cos^{-1}(-2/3)$ $\sqrt{15.68/3}$ exact
iii	[speed = $\lvert v\cos(180 - \theta)\rvert$ = $\sqrt{15.68/3} \times (2/3)$]	M1	For using 'speed at max. height = horiz. comp. of vel. when string becomes slack'
	Speed at greatest height is 1.52 ms^{-1}	A1	
	$0.3gH = \frac{1}{2}\,0.3(5.6^2 - 1.52...^2)$ Greatest height is 1.48 m	M1 A1 [4]	For using the principle of conservation of energy 40/27 exact
	ALTERNATIVE for (iii) $[0 = 2.286..^2 \times (1\text{-}4/9)\ \text{-}19.6y$, $H = 0.8(1 + 2/3) + y]$ $H = 1.3333.. + 0.1481...\ (4/3 + 4/27)$ Greatest height is 1.48 m (40/27)	 M1 A1	For using $0^2 = \dot{y}^2 - 2gy$ and $H = 0.8\{1 + \cos(180 - \theta)\}+y$
	[$\frac{1}{2}m(2.286...^2 - \text{speed}^2) = mg \times 0.1481....$ $\text{speed}^2 = 2.286..^2 - 19.6 \times 0.1481....$] or [$\frac{1}{2}m(5.6^2 - \text{speed}^2) = mg \times 1.481....$ $\text{speed}^2 = 5.6^2 - 19.6 \times 1.481....$] Speed at greatest height is 1.52 ms^{-1}	 M1 A1	For using the principle of conservation of energy

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